# Local Power of Fixed-T Panel Unit Root Tests with Serially Correlated Errors and Incidental Trends

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# Motivation

- Panel data unit root tests are attractive because they are more powerful

- However, they have their own complications (see e.g. Moon et al (2007))

$$
\begin{aligned}
y_{it} &= a_i + \beta_i t + \zeta_{it}, \\
\zeta_{it} &= \varphi \zeta_{it-1} + u_{it}
\end{aligned}
$$

where

$$
\varphi = 1 - \frac{1}{T\sqrt{N}}
$$

# Research Question

- In this paper we examine the problem for fixed T tests

- Short term serial correlation is also a major factor

- Previous work in the area: Bond et al. (2005), Kruiniger (2008,2009), Madsen (2010), Westerlund(2014)

# Implementation

- Two tests are relative in this framework

| | | |
|---|---|---|
| IV test (De Wachter et al. (2007) | intercepts | **trends** |
| WG test (Kruiniger and Tzavalis (2002) | intercepts | trends |

- Derive local power functions of the IV test for serial correlation and incidental trends

# Contributions

1. The IV test is dominates WG test in terms of power
2. The effects of serial correlation depend on the type of tests and on the deterministic specification
3. The presence of serial correlation does not necessarily mean loss of power
4. The incidental trends problem may not exist in the presence of serial correlation
5. The IV test does not suffer from the incidental trends problem

# Incidental Intercepts

- Consider the AR(1) model:

$$\begin{aligned} y_{it} &= a_i + \zeta_{it} \\ \zeta_{it} &= \varphi \zeta_{it-1} + u_{it}, \qquad i = 1, ..., N \text{ and } t = 1, ..., T \end{aligned}$$

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- Stacked over $i$

$$\begin{aligned} y_i &= a_i e + \zeta_i, \\ \zeta_i &= \varphi\zeta_{i-1} + u_i, \end{aligned}$$

$y_i = (y_{i1}, ..., y_{iT})'$, $e = (1, ..., 1)'$, $\zeta_i = (\zeta_{i1}, ..., \zeta_{iT})'$, $u_i = (u_{i1}, ..., u_{iT})'$
$\zeta_{i-1} = (\zeta_{i0}, ..., \zeta_{iT-1})'$ and also $y_{i-1} = (y_{i0}, ..., y_{iT-1})'$

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- Hypothesis of interest

$$\varphi_N = 1 - \frac{c}{\sqrt{N}}$$

$$\begin{aligned} H_0 &: c = 0 \\ H_1 &: c > 0 \end{aligned}$$

# Individual Intercepts - Assumption

- $\{u_{it}\}$ have $E(u_{it}) = 0$, and are independent and homogeneous across $i$.
- $\{u_{it}\}$ are serially correlated across time but $E(u_{i1}u_{iT}) = 0$.
- The $u_{it}$, are independent of $a_i$ and $y_{i0}$ and all variables have bounded $4 + \varepsilon$ moments.

# Individual Intercepts - The Estimators

- The WG estimator

$$\hat{\varphi}_{WG} = \frac{\sum_{i=1}^{N} y_{i-1}' Q y_i}{\sum_{i=1}^{N} y_{i-1}' Q y_{i-1}}$$

where $Q = I_T - e(e'e)^{-1}e'$.

# Individual Intercepts - The Estimators

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  where $Q = I_T - e(e'e)^{-1}e'$.
- The IV estimator

$$E\left[\sum_{t=1}^{T-p-1} y_{it} u_{i,t+p+1}(\varphi)\right] = 0, \quad \text{or}$$

$$E(y'_{i-1}\Pi_p u_i) = 0$$

  thus

$$\hat{\varphi}_{IV} = \frac{\sum_{i=1}^{N} y'_{i-1} \Pi_p y_i}{\sum_{i=1}^{N} y'_{i-1} \Pi_p y_{i-1}}$$

# Individual Intercepts - Tests

- **Theorem 1:** Under Assumption A and as $N \to \infty$

$$\sqrt{N}\hat{V}_{WG}^{-\frac{1}{2}}\hat{\delta}(\hat{\varphi}_{WG} - 1 - \frac{\hat{b}}{\hat{\delta}}) \xrightarrow{d} N(-ck_{WG}, 1)$$

where

$$\frac{\hat{b}}{\hat{\delta}} = \frac{tr(\Psi\hat{\Gamma})}{\frac{1}{N}\sum_{i=1}^{N} y_{i,-1}' Q y_{i,-1}}, \quad V_{WG} = 2tr((A_{WG}\Gamma)^2),$$

and $A_{WG} = \frac{1}{2}(\Lambda' Q + Q\Lambda - \Psi - \Psi')$ $\quad \hat{\Gamma} = \frac{1}{N}\sum_{i=1}^{N} \Delta y_i \Delta y_i'$

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- **Theorem 2:** Under Assumption A and as $N \rightarrow \infty$

$$\sqrt{N}(\hat{\varphi}_{IV} - 1)\hat{V}_{IV}^{-\frac{1}{2}} \xrightarrow{d} N(-ck_{IV}, 1)$$

where

$$V_{IV} = \frac{2tr((A_{IV}\Gamma)^2)}{tr(\Lambda'\Pi_p\Lambda\Gamma)^2}, A_{IV} = \frac{1}{2}(\Lambda'\Pi_p + \Pi_p'\Lambda)$$

# Individual Intercepts - General Local Power Functions

where

$$k_{WG} = \frac{tr(\Lambda' Q \Lambda \Gamma) + tr(F' Q \Gamma) - tr(\Psi \Lambda \Gamma) - tr(\Lambda' \Psi \Gamma)}{\sqrt{V_{WG}}}$$

and

$$k_{IV} = \frac{1}{\sqrt{V_{IV}}}$$

Then the asymptotic local power function is

$$\Phi(z_a + ck)$$

# Individual Intercepts - Examples

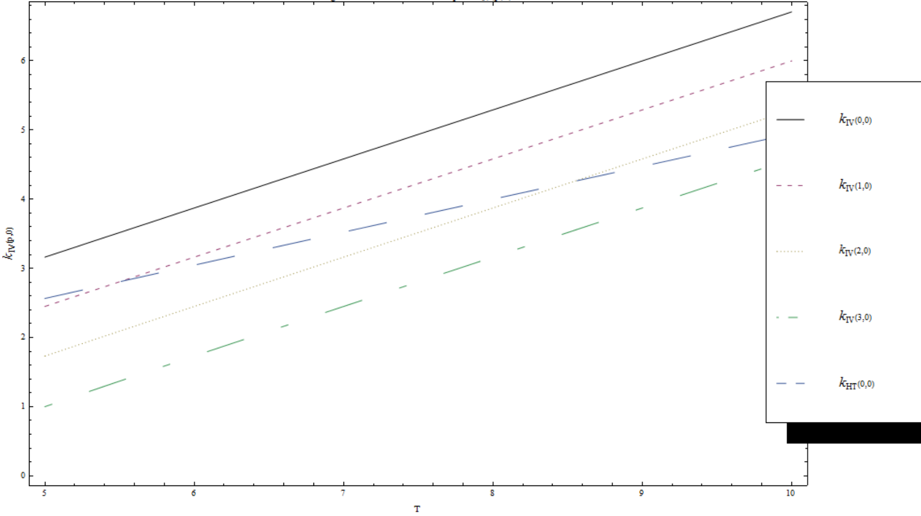
| k | WG | IV |
|---|---|---|
| p=0 | $\frac{\sqrt{3}(T-1)}{\sqrt{T^2-2T-\frac{4}{T}+5}}$ | $\sqrt{\frac{1}{2}(T^2-T)}$ |
| p=1 | $\frac{\sqrt{3}(T^2-3T+2)}{T\sqrt{T^2-6T-\frac{24}{T}+\frac{12}{T^2}+17}}$ | $\sqrt{\frac{T^2}{2}-\frac{3T}{2}+1}$ |
| p=2 | $\frac{\sqrt{3}(T^2-5T+6)}{T\sqrt{T^2-10T-\frac{80}{T}+\frac{60}{T^2}+41}}$ | $\sqrt{\frac{T^2}{2}-\frac{5T}{2}+3}$ |
| p=3 | $\frac{\sqrt{3}(T^2-7T+12)}{T\sqrt{T^2-14T-\frac{196}{T}+\frac{192}{T^2}+77}}$ | $\sqrt{\frac{T^2}{2}-\frac{7T}{2}+6}$ |

and for MA(1) with parameter $\theta$

$$k_{WG} = \frac{(T-2)(T\theta^2-\theta^2+3T\theta-7\theta+T-1)}{2T\sqrt{R_{1,WG}\theta^4+R_{2,WG}\theta^3+R_{3,WG}\theta^2+R_{2,WG}\theta+R_{1,WG}}}$$

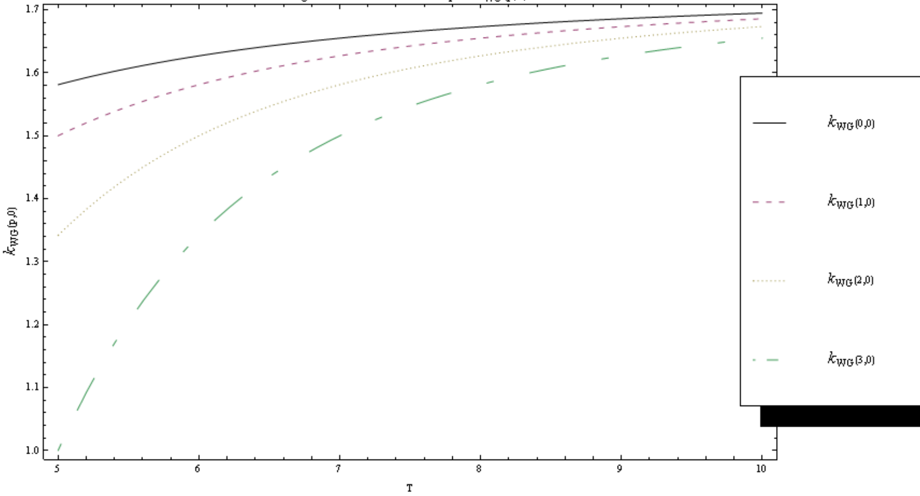
$$k_{IV} = \frac{D_{1,IV}\theta^2+D_{2,IV}\theta+D_{1,IV}}{\sqrt{R_{1,IV}\theta^4+R_{2,IV}\theta^3+R_{3,IV}\theta^2+R_{2,IV}\theta+R_{1,IV}}}$$

Figure 1: The effects of T and p on $k_{IV}(p,0)$



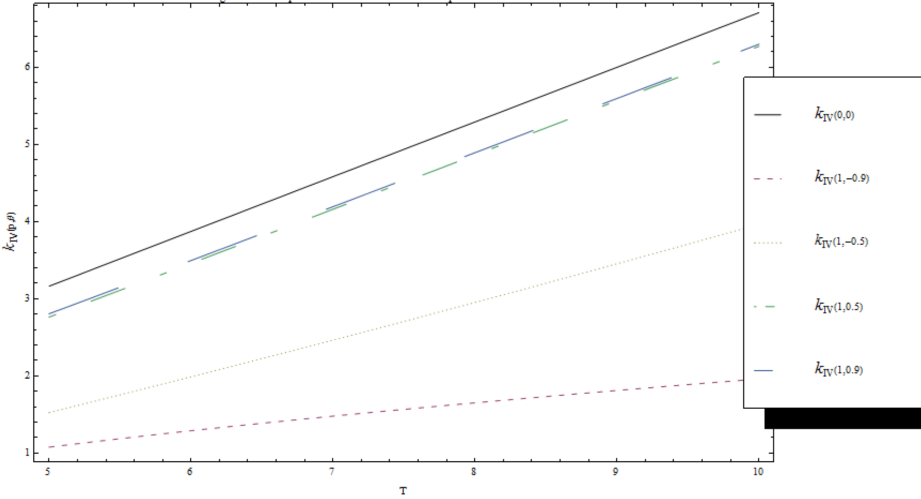
# Individual intercepts - Local Power Functions - WG

Figure 5: The effects of T and p on $k_{WG}(p,0)$

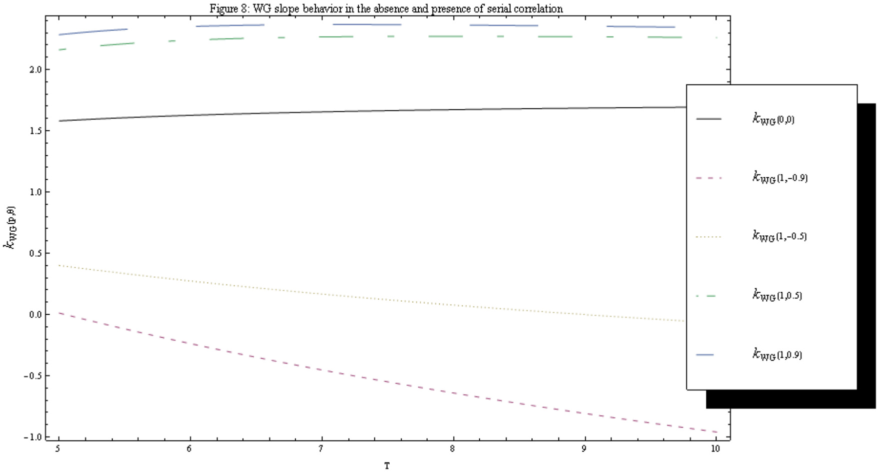


# Individual Intercepts - Local Power Functions - IV

Figure 4: IV slope behavior in the absence and presence of serial correlation



# Individual Intercepts - Local Power Functions - WG



# Incidental Trends - Estimators

- The incidental trends model is

$$\begin{aligned} y_i &= a_i e + \beta_i \tau + \zeta_i, \\ \zeta_i &= \varphi \zeta_{i-1} + u_i \end{aligned}$$

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$$\hat{\varphi}_{WG} = \left( \sum_{i=1}^{N} y_{i-1}' Q y_{i-1} \right)^{-1} \left( \sum_{i=1}^{N} y_{i-1}' Q y_i \right)$$

where $Q = I_T - X(X'X)^{-1}X'$ with $X = [e, \tau]$.

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where $Q = I_T - X(X'X)^{-1}X'$ with $X = [e, \tau]$.

- Taking first differences

$$E(y_{i-1}^{*\prime} \Pi_p^* u_i^*) = 0$$

where $u_i^* = \Delta u_i = (\Delta u_{i2}, ..., \Delta u_{iT})$ and $y_i^* = \Delta y_i = (\Delta y_{i2}, ..., \Delta y_{iT})$ which leads to

$$\hat{\varphi}_{IV} = \left( \sum_{i=1}^{N} y_{i-1}^{*\prime} \Pi_p^* y_{i-1}^* \right)^{-1} \left( \sum_{i=1}^{N} y_{i-1}^{*\prime} \Pi_p^* y_i^* \right)$$

# Incidental Trends - Tests

- **Theorem 3:** Under Assumption A and as $N \to \infty$

$$\sqrt{N}\hat{V}_{WG}^{-\frac{1}{2}}\hat{\delta}(\hat{\varphi}_{WG} - 1 - \frac{\hat{b}}{\hat{\delta}}) \to N(-ck_{WG}, 1)$$

where

$$\frac{\hat{b}}{\hat{\delta}} = \frac{tr(\Phi\hat{\Gamma})}{\frac{1}{N}\sum_{i=1}^{N} y_{i,-1}' Q y_{i,-1}} \text{ and } V_{WG} = vec(\Lambda' Q - \Phi')'\Theta vec(\Lambda' Q - \Phi')$$

$$\text{and } \hat{\Theta} = \frac{1}{N}\sum_{i=1}^{N}\left(vec(\Delta y_i \Delta y_i')vec(\Delta y_i \Delta y_i')'\right)$$

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where $V_{IV} = \frac{2tr((A_{FDIV}\Theta)^2)}{tr(\Lambda^{*\prime}\Pi_p^*\Lambda^*\Theta)^2}$, $A_{IV} = \frac{1}{2}(\Lambda^{*\prime}\Pi_p^* + \Pi_p^{*\prime}\Lambda^*)$ and $\Theta = 2\Gamma_1 - \Gamma_2 - \Gamma_2'$ where $\Gamma_1 = E(u_i u_i')$ and $\Gamma_2 = E(u_i u_{i-1}')$.

# Individual Trends - Local Power Functions

where for the WG test

$$k_{WG} = \frac{tr(\Lambda' Q \Gamma) + tr(F' Q \Gamma) - tr(\Phi \Lambda \Gamma) - tr(\Lambda' \Phi \Gamma)}{2tr((A_{WGT}\Gamma)^2)}$$

and for the IV test

$$k_{IV} = \frac{1}{\sqrt{V_{IV}}}$$

Then

**Table**: Values of slope parameter $p$.

| | | | T=7 | | |
|---|---|---|---|---|---|
| $\theta$ | -0.9 | -0.5 | 0.0 | 0.5 | 0.9 |
| $k_{IV}$ | 0.862 | 0.896 | 1.264 | 1.186 | 1.179 |
| $k_{WG}$ | 0.694 | 0.466 | 0.00 | -0.212 | -0.248 |
| | | | T=10 | | |
| $\theta$ | -0.9 | -0.5 | 0.0 | 0.5 | 0.9 |
| $k_{IV}$ | 1.160 | 1.229 | 1.750 | 1.989 | 2.008 |
| $k_{WG}$ | 1.042 | 0.645 | 0.00 | -0.216 | -0.248 |

# Large T

- If we assume T to be asymptotic the hypothesis of interest becomes

$$\varphi_{NT} = 1 - \frac{c}{T\sqrt{N}}$$

**Theorem 5:** If $T, N \to \infty$ jointly and the following condition holds: $\sqrt{N}/T \to 0$.

$$T\sqrt{N}(\sqrt{2})^{-1}(\hat{\varphi}_{IV} - 1) \xrightarrow{d} N(-c\frac{1}{\sqrt{2}}, 1), \qquad \text{and}$$

$$T\sqrt{N}(\sqrt{3})^{-1}\hat{\delta}_{WG}\left(\hat{\varphi}_{WG} - 1 - \frac{\hat{b}}{\hat{\delta}}\right) \xrightarrow{d} N(-c0, 1),$$

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- Table 2: Slopes of large-T tests.

| IV | MPP | LLC/HT | SGLS | IPS | WG |
|---|---|---|---|---|---|
| $1/\sqrt{2}$ | $1/\sqrt{2}$ | $(3/2)\sqrt{(5/51)}$ | $1/\sqrt{3}$ | 0.282 | 0 |

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- In the presence of incidental trends

$$k_{IV} = 0$$
$$k_{WG} = 0$$

# Heterogeneous Alternatives

For alternatives of the form

$$\varphi_{Ni} = 1 - \frac{c_i}{\sqrt{N}}$$

the hypothesis of interest is

$$\begin{aligned} H_0 &: \quad c_i = 0, \ \text{for all } i \\ H_1 &: \quad c_i > 0, \ \text{for some } i \end{aligned}$$

where

- $c_i$ are $i.i.d.$ with support in a subset of a bounded interval $[0, M_c]$, for some $M_c \geq 0$.

Then, only the mean of $c_i$ affects power, i.e.

$$\sqrt{N}(\hat{\varphi}_{IV} - 1)\hat{V}_{IV}^{-\frac{1}{2}} \rightarrow N(-E(c_i)k_{IV}, 1)$$

# Monte Carlo Simulations

| Intercepts | | | | | Trends | | | | |
|---|---|---|---|---|---|---|---|---|---|
| N | 50 | 200 | 1000 | Theory | N | 50 | 200 | 1000 | Theory |
| $\theta = -0.5$ | | | | | $\theta = -0.5$ | | | | |
| IV | 0.285 | 0.444 | 0.567 | **0.793** | IV | 0.039 | 0.050 | 0.050 | **0.498** |
| WG | 0.057 | 0.068 | 0.087 | **0.069** | WG | 0.093 | 0.108 | 0.107 | **0.158** |
| $\theta = 0$ | | | | | $\theta = 0$ | | | | |
| IV | 0.997 | 0.997 | 0.998 | **0.998** | IV | 0.050 | 0.051 | 0.048 | **0.760** |
| WG | 0.220 | 0.321 | 0.414 | **0.500** | WG | 0.169 | 0.122 | 0.089 | **0.050** |
| $\theta = 0.5$ | | | | | $\theta = 0.5$ | | | | |
| IV | 0.979 | 0.988 | 0.993 | **0.994** | IV | 0.031 | 0.038 | 0.045 | **0.900** |
| WG | 0.388 | 0.57 | 0.678 | **0.730** | WG | 0.217 | 0.135 | 0.070 | **0.033** |

# Conclusions

- The IV test is **always better** than the WG test and better than the HT test
- The effect of serial correlation is **case specific**
- Local power is possible in a fixed T therefore theoretically **solving** the incidental trends problem
- Power comes either from the **presence of serial correlation** or the use of **double differences**
- Non-trivial power **vanishes** when T is asymptotic