Discrete Choice Models of Monetary Policy: Introducing the Tempered Ordered Probit Model*

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Abstract

We propose a Tempered Ordered Probit (TOP) model. Our contribution lies in not only explicitly accounting for an excessive number of observations in a given choice category - as is the case in the standard literature on inflated models; rather, we introduce a new econometric model which nests the recently developed Middle Inflated Ordered Probit (MIOP) models of Brooks, Harris, and Spencer (2012) and Bagozzi and Mukherjee (2012) as a special case, and further, can be used as a specification test of the MIOP, where the implicit test is described as being one of symmetry versus asymmetry.

Keywords: Monetary policy committee, voting, discrete data, ordered models, inflated outcomes.

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1 Introduction

Recent advances in discrete choice modeling have seen the development of so-called inflated models. Such developments have been motivated by the observation that in certain discrete choice situations, a large proportion of empirical observations fall into one particular choice category, such that the category with the excess of observations appears ‘inflated’ relative to the others. In this paper, we add to this growing strand of literature by proposing a Tempered Ordered Probit (TOP) model. Our contribution lies not only in explicitly accounting for an excessive number of observations in a given choice category - as is the case in the standard literature on inflated models; rather, we introduce a new econometric model which nests the recently developed Middle Inflated Ordered Probit (MIOP) models of Brooks, Harris, and Spencer (2012) and Bagozzi and Mukherjee (2012) as a special case, and which further, can be used as a specification test of the MIOP, where the implicit test is described as being one of symmetry versus asymmetry.\(^1\)

Our model is then used to exploit a panel dataset containing the votes of Bank of England Monetary Policy Committee (MPC) members. In our application, which models members’ interest-rate choices, we simultaneously allow for a trichotomous ordered probit equation capturing an economic conditions equation à la Taylor (1993), coupled with a set of direction specific binary probit ‘inertia’ equations which gauge the propensity of MPC members to change or not change the interest rate. Our model explicitly accounts for the inflated number of votes to leave the short-term interest rate unchanged, accounting for no-change decisions such as those arising from policy makers following a ‘wait and see’ policy due to economic uncertainty. Repeated observations for each committee member allow us to condition on the presence of any unobserved heterogeneity pertaining to individual members. However, unlike in Brooks, Harris, and Spencer (2012), where the binary decision to change or not change the policy rate was assumed to be identical irrespective of its direction (e.g. up or down), this paper relaxes this assumption. As noted above, a significant corollary of this innovation is that the TOP model can be used as a specification test of the MIOP model. In the context of our application, the implicit test is one of symmetry versus asymmetry in the inertia equation across the alternatives of inertia compared to up, and inertia compared to down.

Our findings suggest that members are indeed characterized by quite different propensities to

\(^1\)The MIOP model is itself an extension of the Zero-Inflated Ordered Probit Model (Harris and Zhao 2007), where the probability-augmented outcome is not necessarily at one end of the choice spectrum, but in the middle. This implies that for a MIOP model characterised by ordered framework with three choices, the middle category is ‘inflated’.
leave the interest rate unchanged, which in turn represents asymmetric attitudes towards, for instance, economic uncertainty and the changeability of interest rates. This in turn implies that a TOP modelling strategy is preferable to a MIOP one.

2 Discrete-Choice Approaches to Monetary Policy

A number of empirical studies have applied limited dependent variable techniques to modelling monetary policy decisions. A useful starting point, and indeed one that has found much favour in the empirical literature, is the standard ordered probit (OP) model. In such literature, monetary policy decisions are typically coded to reflect decisions to relax, leave unchanged, or tighten policy. Gerlach (2007) for instance models the short term-interest rate setting behavior of the European Central Bank by using the ECB’s *Monthly Bulletin* to inform the choice of explanatory variables. However, it is noteworthy that the application of the OP model is by no means restricted to the using the short-term interest rate to model monetary policy decisions, an approach exemplified in Xiong (2012) who estimates the policy stance of the People’s Bank of China (PBC). As no single instrument best captures the policy stance of the PBC, the author creates a policy stance index which is subsequently discretized and employed as the dependent variable. The vast majority of papers nevertheless focus on datasets where the short-term interest rate is the key instrument of monetary policy.

Other contributions have estimated models within a dynamic setting, which significantly complicates estimation. Eichengreen, Watson, and Grossman (1985) model the setting of the bank rate by the Bank of England in the interwar gold standard period using a dynamic probit model. Davutyan and Parke (1995) extend this approach by applying a dynamic probit model to the setting of the bank rate in the period prior to World War I. Hamilton and Jorda (2002) propose a different approach to modelling the US federal funds target rate over the period from 1984 to 2001. Specifically, they extend the autoregressive conditional duration model (Engle and Russell 1997; Engle and Russell 1998) to model the likelihood that the target rate will change tomorrow, given the available information set today. Significantly, the Hamilton and Jorda (2002) model also includes an ordered probit component. Dolado and Maria-Dolores (2002) provide an alternative in the framework of a marked-point-process approach by applying a sequential probit model to understand the interest rate policy of the Bank of Spain for the period 1984 to 1998. The same authors (Dolado and Maria-Dolores 2005) also employ an ordered probit approach to study the interest rate setting behaviour
of four European central banks and the US Federal Reserve.\(^2\) Other approaches have used unordered approaches, such as Allen, Bray, and Seaks (1997), Tootell (1991b) and Tootell (1991a) who employ multinomial logit analysis to model aspects of Federal Reserve interest rate setting behavior.

Within the context of our own empirical application, a number of contributions have taken advantage of the information contained in the voting records of monetary policy committees with a view to attempting to account for differences in members’ voting behavior or predicting future monetary policy decisions. Indeed, Tootell (1991b) and Tootell (1991a) employ a multinomial logit model to test for differences in the voting behavior of FOMC members.\(^3\) In contrast, Gerlach-Kristen (2004) uses a standard ordered probit framework to demonstrate that voting record information can be used to predict future changes in the Bank of England’s short-term interest rate. This is achieved through using a measure called skew, which proxies for the extent to which MPC members disagree with each other at a given meeting. As the current paper also exploits the MPC’s voting record - in our case, a panel of MPC members’ votes on the short term interest-rate - we find it fruitful to expound our formal discussion of the TOP model in such a context. Moreover, as the foundation of our formal analysis is the (panel) ordered probit model, we use this as starting point.

\(^2\)It is also possible to consider an interval regression approach. Essentially this is very similar to the ordered probit approach, except that one now: makes a decision regarding the quantitative value of the cut-points (for example, it might be deemed to be 1.75% in the choice between the two policy rates of 1.5 and 2%); once such assumptions are made, it is now also possible to estimate the variance of \(y\). That is here, we take advantage of the magnitude of the rate choices (1%, 2%, 2.5%, and so on). Not surprisingly, these two approaches tend to yield very similar results.

\(^3\)This paper is also related to a literature which is geared towards explaining the voting behaviour of members of the United States FOMC. As we generally model MPC members’ votes as a function of the economic environment, it falls into what Meade and Sheets (2005) label the ‘reaction function’ camp (Tootell 1991b; Tootell 1991a), and not the ‘partisan theory of politics’ genus of studies (Belden 1989; Havrilesky and Schweitzer 1990; Havrilesky and Gildea 1991). For example, Tootell (1991b) tests the hypothesis that District Bank Presidents set policy according to regional, as opposed to national economic conditions. No evidence to support this claim is found, although evidence to the contrary is found by Meade and Sheets (2005). In a further paper, Tootell (1991a) tests, but fails to find evidence, to support the hypothesis that Federal Reserve Bank Presidents vote more ‘conservatively’ than members of the Board of Governors. In both contributions (Tootell 1991b; Tootell 1991a) Greenbook estimates of GDP growth and inflation are used as covariates. Given that the economy is influenced with lags by monetary policy, it follows that FOMC members’ votes are most likely determined by their expectations of inflation and GDP growth (as opposed to their current, or past, values).
2.1 The (Panel) Ordered Probit

Consider a situation where we have repeated observations on members of a monetary policy committee. Each MPC member \( i \) is envisaged to have an underlying, unobserved, propensity to vote, for a desired rate in meeting \( t \), denoted \( y^*_it \). This will be driven by a set of economic conditions prevailing at time \( t \) to the member, \( x_{it} \) with unknown weights \( \beta \) and a random disturbance term \( \varepsilon_{it} \) such that

\[
y^*_it = x_{it}'\beta + \varepsilon_{it}.
\]

This unobserved index will translate into votes for a rate decrease \((y = -1)\), no-change \((0)\) and increase \((y = 1)\) according to the relationship between \( y^* \) and boundary parameters, \( \mu \)

\[
y = \begin{cases} 
-1 & \text{if } y^* < \mu_0 \\
0 & \text{if } \mu_0 \leq y^* < \mu_1 \\
1 & \text{if } y^* \geq \mu_1 
\end{cases}
\]

where, for identification, \( \mu_0 \) is normalised to 0 (or equivalently, there is no constant in \( x \)) and where \( V(\varepsilon_{it}) = 1 \), also for identification. Under the usual assumption of normality, this results in probabilities of each observed state of

\[
\Pr(y_{it}) = \begin{cases} 
-1 & = \Phi(-x_{it}'\beta) \\
0 & = \Phi(\mu_1 - x_{it}'\beta) - \Phi(x_{it}'\beta) \\
1 & = 1 - \Phi(\mu_1 - x_{it}'\beta)
\end{cases}
\]

Several authors have based analyses on such a set-up; and, as in this paper, some studies have utilized information contained in the MPC’s voting record. For instance, Harris and Spencer (2009) adopt a related approach to the current paper, using a similar panel data set of MPC members’ votes and discrete choice methods. Simple ordered probability models are estimated although the focus is mainly on the inherent differences between the voting behaviour of ‘internal’ and ‘external’ MPC members.\(^4\). The reason for such a favoured approach is primarily based upon and/or justified by, the empirical regularity that both observed policy rate changes, and votes for changes thereof, are all invariably in the order of \( \pm 25 \) or 50 basis points. Indeed, the latter are quite rare, and therefore would be hard to model, therefore the decision is very often converted into a simple up/down/no change

\(^4\)The internal-external distinction is also followed in Gerlach-Kristen (2003) who shows that disagreements between members of the Bank’s MPC typically constitute the rule, and not the exception. The paper provides more of a descriptive overview of MPC voting behavior.
choice. Indeed, for the expositions below, we will also (unless otherwise stated) assume a three choice scenario of: up (1); no-change (0); and down (−1), possibly augmented, for example to additionally include unobserved effects in equation (1). Such an approach, however, does not address the relative preponderence of no change decisions.

2.2 Middle-Inflated Models

Thus far, there is a limited discrete-choice literature attempting to address the empirical regularity of an “excess” of observations corresponding to no-change in the interest rate. Brooks et al. (2012) address this issue by using a two-stage decision based approach. Their formal starting point is an underlying latent variable, which variable represents an overall propensity to choose the inflated category over any other, and therefore translates into an “observed” binary outcome. This latent variable $q^*$, can be thus labelled an “inertia” (or “splitting”) equation, and is assumed to be a linear in parameters ($\beta_s$) function of a vector of observed characteristics $x_s$ and a random error term $\varepsilon_s$

$$q^* = x'_s\beta_s + \varepsilon_s.$$ (4)

A two-regime scenario is then proposed such that for observations in regime $q = 0$, the inflated (no-change) outcome is observed; but for those in $q = 1$ any of the possible outcomes in the choice set $\{-1, 0, 1\}$ which includes the outcome with an excess of observations. Of course we never observe membership of either regime ($q = 0, q = 1$), and must rely on data to identify this.

For units in regime $q = 1$, an underlying latent variable $y^*$ is specified as a linear in parameters function of a vector of observed characteristics $x_y$ (with no constant), with unknown weights $\beta_y$ and a random normally disturbance term $u_y$ thus

$$y^* = x'_y\beta_y + \varepsilon_y.$$ (5)

For individuals in this regime, outcome probabilities are determined by an OP model. Thus

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5 Using a linear random parameters model, Besley, Meads, and Surico (2008) demonstrate that although MPC decisions are characterized by voter heterogeneity, the differences in reactions to the inflation forecast and output gap based on a member’s type and career backgrounds are insignificant. This is in contrast to Harris and Spencer (2009) who find that internal and external members react very differently to forecasts of inflation and output.

6 That is, even in a continually changing economic environment, both the policy rate, and votes thereof, are dominated by these no-change observations.
under the
\[
\Pr(y_{it} = -1 | z_{it}, x_{it}) = \Phi (x'_s \beta_s) \times \Phi \left( \mu_0 - x'_y \beta_y \right)
\]
\[
\Pr(y_{it} = 0 | z_{it}, x_{it}) = \left[ 1 - \Phi (x'_s \beta_s) \right] + \Phi (x'_s \beta_s) \times \left[ \Phi \left( \mu_1 - x'_y \beta_y \right) - \Phi \left( \mu_0 - x'_y \beta_y \right) \right]
\]
\[
\Pr(y_{it} = 1 | z_{it}, x_{it}) = \Phi (x'_s \beta_s) \times \left[ 1 - \Phi \left( \mu_1 - x'_y \beta_y \right) \right]
\]\n(6)

where \(\Phi\) denotes the cumulative distribution function of the standardised normal distribution, and \(\mu_j\) are the \(J - 1\) usual boundary parameters. In this way, the probability of outcome 1 has been ‘inflated’. Thus to observe a \(y_{it} = 0\) outcome we require either that \(q = 0\); or jointly that \(q = 1\) and that \(\mu_0 < y^* \leq \mu_1\). Observationally equivalent no-change outcomes, can hence arise from two distinct sources. In terms of exclusion restrictions Brooks et al. (2012) propose that the variables entering \(x_s\) should be Taylor-rule type ones, whereas those in \(x_y\) should be more institutional in nature, and include proxies for risk and uncertainty.

### 2.3 The Tempered Ordered Probit (TOP) Model

It is possible to consider a further refinement to the OP model. As with the usual OP set-up described above, let each observational unit have a propensity to vote, for a desired rate, \(y^*\). This can be assumed to again be a function prevailing economic conditions \(x_y\) with unknown weights \(\beta_y\) and a random disturbance term \(u_y\), as per equation (1). However, to allow for the observed build-up of no-change observations, the movement propensities (that is the up and down ones) are both tempered by two further equations that allow observations with either of these propensities to still choose no-change, presumably as a function of proxies for uncertainty and institutional factors, such as in \(x_s\) above. We term this model, the Tempered Ordered Probit (TOP) model. Clearly it would be possible to allow a different variables affect the tempering on the up and down propensities, but this seems difficult to justify on a priori grounds. Thus we assume that there is one block of variables \((x_s)\) that drives both of these tempering equations.

Explicitly, to incorporate uncertainty into the propensities for vote decreases and increases, respectively, requires specification of two further latent variables, \(d^*\) and \(u^*\). Thus for observations that have an down propensity, whether they actually choose this outcome or alternatively opt for a no-change outcome, will be determined by the former, and will be the result of a binary (yes/no) decision for this observation. Let this process be determined by an equation of the form
\[
d^* = x'_s \beta_d + \varepsilon_d
\]
(7)
then, under the assumption of normality, conditional on the member having a down propensity, the probability of a vote decrease will be

$$\Pr (\text{decrease} | \text{down propensity}) = \Phi (x_s' \beta_d)$$  \hspace{1cm} (8)$$

and, by symmetry, for no-change

$$\Pr (\text{no – change} | \text{down propensity}) = \Phi (-x_s' \beta_d).$$  \hspace{1cm} (9)$$

Similarly for members who have an up propensity, on the basis of the latent propensity equation of

$$u^* = x_s' \beta_u + \varepsilon_u$$  \hspace{1cm} (10)$$

the probability of them voting for rate increase will be given by

$$\Pr (\text{increase} | \text{up propensity}) = \Phi (x_s' \beta_u)$$  \hspace{1cm} (11)$$

and for no-change

$$\Pr (\text{no – change} | \text{up propensity}) = \Phi (-x_s' \beta_u).$$  \hspace{1cm} (12)$$

Under independence, the overall probabilities of vote decreases, no-change and increases, will therefore be

$$\Pr (y) = \begin{cases} 
-1 = \Phi (\mu_0 - x_y' \beta_y) \times \Phi (x_s' \beta_d) \\
0 = \left[ \Phi (\mu_1 - x_y' \beta_y) - \Phi (\mu_0 - x_y' \beta_y) \right] + \\
\left[ \Phi (\mu_0 - x_y' \beta_y) \times \Phi (-x_s' \beta_d) \right] + \\
\left[ (1 - \Phi (\mu_1 - x_y' \beta_y)) \times \Phi (-x_s' \beta_d) \right] \\
1 = \left[ 1 - \Phi (\mu_1 - x_y' \beta_y) \right] \times \Phi (x_s' \beta_u) 
\end{cases}$$  \hspace{1cm} (13)$$

In this way, the empirical regularity of an “excess” of no-change votes is allowed for by the additional terms of $\left[ \Phi (\mu_0 - x_y' \beta_y) \times \Phi (-x_s' \beta_d) \right]$ and $\left[ (1 - \Phi (\mu_1 - x_y' \beta_y)) \times \Phi (-x_s' \beta_d) \right]$ in equation (13), which represents member uncertainty.

### 2.4 A Specification Test for the Inflated Ordered Probit Model

There is an interesting empirical issue of whether the down and up propensities were tempered to the same extent, that is whether $\beta_d = \beta_u$. Indeed, such a simple linear parameter
Enforcing this in equation (13 results in

\[
\begin{align*}
\Pr(y) &= \begin{cases} 
-1 & \Phi(\mu_0 - x'_y \beta_y) \times \Phi(x'_s \beta_s) \\
0 & \Phi(\mu_1 - x'_y \beta_y) \times \Phi(\mu_0 - x'_y \beta_y) + [\Phi(\mu_0 - x'_y \beta_y) \times \Phi(-x'_y \beta_s)] + [1 - \Phi(\mu_1 - x'_y \beta_y) \times \Phi(-x'_y \beta_s)] \\
1 & [1 - \Phi(\mu_1 - x'_y \beta_y)] \times \Phi(x'_s \beta_s).
\end{cases}
\end{align*}
\]

Focusing on \(\Pr(y = 0)\) of equation (14) and re-arranging will prove instructive

\[
\begin{align*}
\Pr(y = 0) &= \Phi(\mu_1 - x'_y \beta_y) - \Phi(\mu_0 - x'_y \beta_y) + [\Phi(\mu_0 - x'_y \beta_y) \times \Phi(-x'_y \beta_s)] + [1 - \Phi(\mu_1 - x'_y \beta_y)] \times [1 - \Phi(x'_s \beta_s)] \\
&= \Phi(\mu_1 - x'_y \beta_y) - \Phi(\mu_0 - x'_y \beta_y) + [\Phi(\mu_0 - x'_y \beta_y) \times \Phi(-x'_y \beta_s)] + [1 - \Phi(\mu_1 - x'_y \beta_y)] \times [1 - \Phi(x'_s \beta_s)] \\
&= \Phi(\mu_1 - x'_y \beta_y) - \Phi(\mu_0 - x'_y \beta_y) + [1 - \Phi(\mu_1 - x'_y \beta_y) \times \Phi(-x'_y \beta_s)] \\
&\quad - [1 - \Phi(\mu_1 - x'_y \beta_y) \times \Phi(\mu_0 - x'_y \beta_y)] \times [\Phi(x'_s \beta_s)] \\
&= \Phi(\mu_1 - x'_y \beta_y) - \Phi(\mu_0 - x'_y \beta_y) + [1 - \Phi(\mu_1 - x'_y \beta_y)] \times [\Phi(x'_s \beta_s)] \\
&\quad - [\Phi(\mu_1 - x'_y \beta_y) \times \Phi(\mu_0 - x'_y \beta_y)] \times [\Phi(x'_s \beta_s)] \\
&= [1 - \Phi(x'_s \beta_s)] + [(\Phi(\mu_1 - x'_y \beta_y) - \Phi(\mu_0 - x'_y \beta_y))] \times \Phi(x'_s \beta_s)
\end{align*}
\]

Or equivalently, by re-arranging the \(\Pr(y = 0)\) expression as 1 minus the \(\Pr(y = -1)\) and \(\Pr(y = 1)\) terms of equation (14) we have

\[
\begin{align*}
\Pr(y = 0) &= 1 - \Phi(\mu_0 - x'_y \beta_y) \times \Phi(x'_s \beta_s) - [(1 - \Phi(\mu_1 - x'_y \beta_y)) \times \Phi(x'_s \beta_s)] \\
&= \Phi(x'_s \beta_s) + [1 - \Phi(x'_s \beta_s)] \\
&\quad - [\Phi(\mu_0 - x'_y \beta_y) \times \Phi(x'_s \beta_s)] - [(1 - \Phi(\mu_1 - x'_y \beta_y)) \times \Phi(x'_s \beta_s)] \\
&= [1 - \Phi(x'_s \beta_s)] + [1 - \Phi(\mu_0 - x'_y \beta_y) - (1 - \Phi(\mu_1 - x'_y \beta_y))] \times \Phi(x'_s \beta_s) \\
&= [1 - \Phi(x'_s \beta_s)] + [\Phi(\mu_1 - x'_y \beta_y) - \Phi(\mu_0 - x'_y \beta_y)] \times \Phi(x'_s \beta_s)
\end{align*}
\]
yielding the rewritten restricted probabilities as

\[
\begin{align*}
\Pr (y) &= \begin{cases} 
-1 &= \Phi \left( \mu_0 - \mathbf{x}_y \beta_y \right) \times \Phi \left( \mathbf{x}_c \beta_d \right) \\
0 &= \left[ 1 - \Phi \left( \mathbf{x}_s \beta_s \right) \right] + \left[ \Phi \left( \mu_1 - \mathbf{x}_y \beta_y \right) - \Phi \left( \mu_0 - \mathbf{x}_y \beta_y \right) \right] \times \Phi \left( \mathbf{x}_s \beta_s \right) \\
1 &= \left[ 1 - \Phi \left( \mu_1 - \mathbf{x}_y \beta_y \right) \right] \times \Phi \left( \mathbf{x}_s \beta_u \right)
\end{cases}
\end{align*}
\] (15)

A comparison of equations (6) and (14) shows that the restricted form of the TOP model is identical to that of the IOP one. That is, even though different inherent sequences in the choice process are used to justify both models, they are equivalent under a simple set of parameter restrictions. In this way the TOP model can be used as a specification test of the IOP, where the implicit test is one of symmetry versus asymmetry in the inertia equation across the alternatives of inertia compared to up, and inertia compared to down. An appropriate testing procedure would appear to be a likelihood ratio test of TOP versus IOP, with degrees of freedom the rank of \( \mathbf{x}_s \).

\section*{2.5 Beyond Trichotomous Choice}

It is possible, probable even, that the researcher will be faced with a five, or more outcomes; and moreover be faced with cell sizes that do not necessarily suggest collapsing of cells. In this instance, we would suggest, to maintain the nesting of the IOP model, that the “first” decision is one of: large-increase, small-increase, no-change, small-decrease and finally, large-decrease. However, again because of the hypothesised inertia in these choice decisions, these (change) propensities will again all be tempered. Due to the inertia, and the apparent pull towards “zero”, a propensity for small-increase will be tempered by the binary decision of small-increase or no-change (that is, a movement from here to large-increase is not entertained).

Conversely, what of those in a large-increase propensity? This decision could be tempered by the binary choice of both small-increase or no-change. Although this is likely to vary by application, we suggest here that an appropriate choice-set would be between large-decrease and no-change. There are two viable alternatives to this: 1) consider the choice-set as large-increase, small-increase and no-change: this would both require a further OP equation and therefore would not (obviously) nest the restricted IOP model, and would essentially put extra probability mass into all of the small-increase (decrease) and no-change categories (which may, however, be warranted by the particular application). 2) Consider the choice-set as large-increase and small-increase: again this does not nest the IOP as a special case, and moreover would only serve to put extra mass into the small-increase (decrease) categories.
However, as is obvious, this generic TOP set-up (for more than three outcomes) offers the applied a researcher a very rich variety of options.

Under the assumption of tempering only to *no-change*, the TOP model here would have probabilities of the form

\[
\begin{aligned}
\Pr (y) &= \begin{cases} 
-2 &= \Phi (\mu_0 - x'_y \beta_y) \times \Phi (x'_d \beta_{d,-2}) \\
-1 &= \left[ \Phi (\mu_1 - x'_y \beta_y) - \Phi (\mu_0 - x'_y \beta_y) \right] \times \Phi (x'_d \beta_{d,-1}) \\
&\quad + \left[ \Phi (\mu_2 - x'_y \beta_y) - \Phi (\mu_1 - x'_y \beta_y) \right] \times \Phi (-x'_d \beta_{d,-1}) + \\
0 &= \left[ \Phi (\mu_0 - x'_y \beta_y) \times \Phi (-x'_d \beta_{d,-2}) \right] \times \Phi (-x'_d \beta_{d,-2}) + \\
&\quad \left[ \Phi (\mu_2 - x'_y \beta_y) - \Phi (\mu_1 - x'_y \beta_y) \right] \times \Phi (-x'_u \beta_{u,1}) + \\
1 &= \left[ \Phi (\mu_3 - x'_y \beta_y) - \Phi (\mu_2 - x'_y \beta_y) \right] \times \Phi (x'_u \beta_{u,1}) \\
2 &= \left[ 1 - \Phi (\mu_3 - x'_y \beta_y) \right] \times \Phi (x'_u \beta_{u,2}) \\
\end{cases}
\end{aligned}
\]

where \( \beta_{d,-2} \) are the coefficients in the binary tempering equation for *large-decrease* propensites (where the choice-set is *large-decrease* or *no-change*); and so on.

Several hypothesis tests would be of interest here. Firstly, that there may be only one tempering decision in say, just the up-propensities, \( H_0 : \beta_{d,-2} = \beta_{d,-1} \); or only a single tempering decision in both of the up- and down-propensites, \( H_0 : \beta_{d,-2} = \beta_{d,-1} \) and \( \beta_{u,-2} = \beta_{u,-1} \). As before, an obvious one would be a single tempering decision, \( H_0 : \beta_{d,-2} = \beta_{d,-1} = \beta_{d,-2} = \beta_{d,-1} \), which would test the general TOP model versus the (restricted) IOP one. As these are all simple tests of parameter restrictions, Likelihood ratio tests would appear to be an obvious choice.

### 2.6 Unobserved Heterogeniety

However, the random disturbances of equations (1), (7) and (10) all relate to the same unit of observation, and therefore correlations between these are very likely. That is, for example, the unobservables driving the up/down/no-change propensity (of equation (1)), are likely to be correlated with those which then drive the up/no-change ones for those members having a “up” propensity. However, given that any member in anytime period can only be in any one of the “down” and “up” propensities, there can be no correlations across \( \varepsilon_u \) and \( \varepsilon_d \), but \( \varepsilon_y \) and \( \varepsilon_d \) will be correlated with correlation coefficient \( \rho_{yd} \); and similarly \( \varepsilon_y \) and \( \varepsilon_u \) with coefficient \( \rho_{yu} \). Now the joint probabilities of equation (13) are no longer products of
univariate probabilities, but bivariate normal ones such that

$$\Pr(y) = \begin{cases} -1 & = \Phi_2 \left( \mu_0 - x'_y \beta_y, x'_s \beta_s; -\rho_{yd} \right) \\ 0 & = \left[ \Phi \left( \mu_1 - x'_y \beta_y \right) - \Phi \left( \mu_0 - x'_y \beta_y \right) \right] + \Phi_2 \left( \mu_0 - x'_y \beta_y, -x'_s \beta_d; \rho_{yu} \right) + \Phi_2 \left( x'_y \beta_y - \mu_1, -x'_s \beta_d; -\rho_{yd} \right) \\ 1 & = \left[ \Phi_2 \left( x'_y \beta_y - \mu_1, x'_s \beta_s; \rho_{yu} \right) \right] \end{cases} \quad (17)$$

where $\Phi_2 (a, b; \rho)$ represents the standardized bivariate Normal density with upper integration limits $a$ and $b$ with correlation $\rho$.

Treating each observation as an independent random draw, the estimation of probabilities given by equation (17) is obtained by maximizing the log-likelihood function $L(\theta)$ with respect to the parameter vector $\theta = (\beta', \mu', \rho)'$. So, for $J = 3$ vote outcomes and each member $i$ being observed for $t = 1, \ldots, T_i$ time periods, we have

$$\ln L(\theta) = \sum_{i=1}^{N} \sum_{t=1}^{T_i} \sum_{j=-1}^{1} d_{ijt} \ln \left[ \Pr \left( y_{it} = j | X \right) \right] \quad (18)$$

where $d_{ijt}$ is the indicator function such that

$$d_{ijt} = \begin{cases} 1 \text{ if individual } i \text{ chooses outcome } j \text{ in meting } t \\ 0 \text{ otherwise.} \end{cases} \quad (19)$$

for $i = 1, \ldots, N; j = -1, 0, 1; t = 1, \ldots, T_i$.

We extend out baseline model in two important ways. Firstly, we introduce additive heterogeneity - or “traditional” unobserved (random) effects - into the two (conditional) up and down propensity equations. Thus equations (10) and (7), respectively become

$$u^* = x'_y \beta_y + \alpha_{iu} + \varepsilon_u \quad (20)$$

and

$$d^* = x'_d \beta_d + \alpha_{id} + \varepsilon_d \quad (21)$$

where the $i$ index on both $\alpha$’s is to make clear that these are observation-varying, but constant over time. As is common in the panel data we will make the assumption that $\alpha_{iu} \sim N(0, \sigma_u^2)$ and $\alpha_{id} \sim N(0, \sigma_d^2)$. That is, even though two members may both be in “up propensity” positions, and conditional on their realisations of $x_s$ they are still likely to have differing

\footnote{This somewhat restrictive simplifying assumption is relaxed below.}
conditional propensities for up and no-change. It is exactly these differing propensities that these unobserved effects will account for.

The second way we extend our baseline model is allow for members to equivalently have differing information sets, or for them to react differently to the same information sets. The information sets that MPC members will be utilising in their voting decisions will be those primarily relating to inflation forecasts (as they are primarily charged with the responsibility of achieving inflation targets), but also to GDP. Thus the approach we adopt here, is a random parameters one where we allow member-specific coefficients on all the Taylor-rule variables in the first propensity equation, equation (1) such that

\[ \beta_i^p = \beta^p + e_i^p \]
\[ \beta_i^{GDP} = \beta^{GDP} + e_i^{GDP} \]

where \( e_i^p \sim N(0, \sigma^2_p) \) and \( e_i^{GDP} \sim N(0, \sigma^2_{GDP}) \).

However, the presence of such unobserved effects complicates evaluation of the resulting likelihood function. Effectively all of these unobserved elements need to be integrated out of the likelihood function. To this end we utilise simulated maximum likelihood techniques, with Halton random. Essentially this entails random draws from the assumed normal distribution(s), which are then entered into equations (20) to (22) and the likelihood evaluated for this particular set of draws. This is undertaken \( r = 1, \ldots, R \) times and the resulting simulated likelihood function is the average of these \( r \) ones over \( R \). However, now due to the dependence across observations arising from the inclusion of these various unobserved effects, the likelihood for an individual is the product of their sequences of individual likelihoods over the \( T_i \) time period that they are observed for. Thus the log-simulated likelihood, \( \ln L(\theta)_s \), becomes

\[ \ln L(\theta)_s = \sum_{i=1}^{N} \ln \left( \frac{1}{R} \sum_{r=1}^{R} \prod_{t=0}^{T_i-1} d_{ijt} \Pr(y_{it,r} = j | \mathbf{X}, r) \right) \]  

(23)

where now \( \theta \) additionally includes the for variance parameters.
3 Empirical Application

In the decade that followed the granting of operational independence to the Bank of England in May 1997, three notable empirical regularities characterized the behavior of the policy rate. First, even with the arrival of new economic information, interest rates were infrequently adjusted. Second, when rates were moved, they were done so in discretized fixed intervals, typically 25 basis point multiples ranging from $-50$ to $+25$ basis points. Third, when rates were adjusted, they moved in a series of small unidirectional 25 basis point steps rather than fewer relatively larger ones.

These stylized regularities also extend to individual votes on the policy rate cast by Bank of England MPC members. In our application, we propose that to explain members’ votes, the confluence of these characteristics warrants a *Tempered Ordered Probit* (TOP) model. Our application models members’ interest-rate choices by simultaneously allowing for a trichotomous ordered probit equation capturing a policy rule à la Taylor (1993), coupled with a set of binary direction specific probit ‘inertia’ equations which gauge the propensity of MPC members to change or not change the interest rate. Our estimation strategy explicitly accounts for the preponderance of votes to leave the interest rate unchanged, accounting for no-change decisions such as those arising from policy makers following a ‘wait and see’ policy due to economic uncertainty. Repeated observations for each committee member allow us to condition on the presence of any unobserved heterogeneity appertaining to individual members. This system of equations not only allows one to ‘inflate’ the probability of a no-change decision on interest rates, thereby allowing such observations to arise from two distinct sources; significantly, it also allows us to test whether members exhibit asymmetric preferences towards economic uncertainty and the changability of interest rates. As mentioned, because the overwhelming majority of votes cast to change interest rates as 25 basis points in magnitude, we assume a *three choice* scenario of: up (1); no-change (0); and down ($-1$). Very little information is thus lost in the process of estimation.

Figure 2 is a graphical representation of the TOP model. 

---

8 Unlike in (Brooks, Harris, and Spencer 2012), where the binary decision to change or not change the policy rate was assumed to be identical irrespective of its direction (e.g. up or down), this paper relaxes this assumption.

9 For some of estimation sample, the nominal interest rate was at the effective zero-lower bound. In this period, the choice-set faced by the MPC members is clearly restricted to one of no-change or up. For this part of the sample, we restrict the choice-set to these alternatives.
## Table 1: MPC members June 1997 - December 2011

<table>
<thead>
<tr>
<th>Member</th>
<th>First Meeting</th>
<th>Last Meeting</th>
<th>Votes cast</th>
<th>Breakdown of votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mervyn King</td>
<td>June 1997</td>
<td>June 2013†</td>
<td>176</td>
<td>23 (13.1) 123 (69.9) 30 (17.0)</td>
</tr>
<tr>
<td>Eddie George</td>
<td>June 1997</td>
<td>May 2003</td>
<td>74</td>
<td>15 (20.3) 50 (67.6) 9 (12.1)</td>
</tr>
<tr>
<td>Howard Davies</td>
<td>June 1997</td>
<td>July 1997</td>
<td>2</td>
<td>0 (0) 2 (100) 0 (0)</td>
</tr>
<tr>
<td>Ian Plenderleith</td>
<td>June 1997</td>
<td>May 2002</td>
<td>61</td>
<td>13 (21.3) 39 (64.0) 9 (14.7)</td>
</tr>
<tr>
<td>John Vickers</td>
<td>June 1998</td>
<td>Sept. 2000</td>
<td>28</td>
<td>7 (25.0) 11 (39.3) 10 (35.7)</td>
</tr>
<tr>
<td>Charles Bean</td>
<td>Oct. 2000</td>
<td>July 2013*</td>
<td>136</td>
<td>22 (16.2) 106 (77.9) 8 (5.9)</td>
</tr>
<tr>
<td>Paul Tucker</td>
<td>June 2002</td>
<td>Feb 2014*</td>
<td>115</td>
<td>10 (8.7) 92 (80) 13 (11.3)</td>
</tr>
<tr>
<td>Andrew Large</td>
<td>Oct. 2002</td>
<td>Jan. 2006</td>
<td>40</td>
<td>1 (2.5) 27 (67.5) 12 (30.0)</td>
</tr>
<tr>
<td>Rachel Lomax</td>
<td>July 2003</td>
<td>June 2008</td>
<td>60</td>
<td>3 (5) 50 (83.3) 7 (11.7)</td>
</tr>
<tr>
<td>John Gieve</td>
<td>Feb. 2006</td>
<td>March 2009</td>
<td>37</td>
<td>10 (27) 21 (56.8) 6 (16.2)</td>
</tr>
<tr>
<td>Willem Buiter</td>
<td>June 1997</td>
<td>May 2000</td>
<td>36</td>
<td>10 (27.8) 9 (25.0) 17 (47.2)</td>
</tr>
<tr>
<td>Charles Goodhart</td>
<td>June 1997</td>
<td>May 2000</td>
<td>36</td>
<td>7 (19.5) 17 (47.2) 12 (33.3)</td>
</tr>
<tr>
<td>De Anne Julius</td>
<td>Sept. 1997</td>
<td>May 2001</td>
<td>45</td>
<td>18 (40.0) 24 (53.3) 3 (6.7)</td>
</tr>
<tr>
<td>Alan Budd</td>
<td>Dec. 1997</td>
<td>May 1999</td>
<td>18</td>
<td>6 (33.3) 6 (33.3) 6 (33.3)</td>
</tr>
<tr>
<td>Sushil Wadhwa†</td>
<td>June 1999</td>
<td>May 2002</td>
<td>37</td>
<td>16 (43.2) 18 (48.7) 3 (8.1)</td>
</tr>
<tr>
<td>Stephen Nickell</td>
<td>June 2000</td>
<td>May 2006</td>
<td>73</td>
<td>23 (31.5) 41 (56.2) 9 (12.3)</td>
</tr>
<tr>
<td>Christopher Alsopp§</td>
<td>June 2000</td>
<td>May 2003</td>
<td>37</td>
<td>18 (48.7) 19 (51.3) 0 (0)</td>
</tr>
<tr>
<td>Kate Barker‡</td>
<td>June 2001</td>
<td>May 2010</td>
<td>109</td>
<td>20 (18.3) 78 (71.6) 11 (10.1)</td>
</tr>
<tr>
<td>Marian Bell</td>
<td>July 2002</td>
<td>May 2005</td>
<td>36</td>
<td>6 (16.7) 26 (72.2) 4 (11.1)</td>
</tr>
<tr>
<td>Richard Lambert</td>
<td>June 2003</td>
<td>Mar. 2006</td>
<td>34</td>
<td>2 (5.9) 27 (79.4) 5 (14.7)</td>
</tr>
<tr>
<td>David Walton§**</td>
<td>July 2005</td>
<td>June 2006</td>
<td>12</td>
<td>2 (16.6) 8 (66.6) 2 (16.6)</td>
</tr>
<tr>
<td>David Blanchflower§</td>
<td>June 2006</td>
<td>May 2009</td>
<td>36</td>
<td>19 (52.8) 16 (44.4) 1 (2.8)</td>
</tr>
<tr>
<td>Timothy Besley§</td>
<td>Sept. 2006</td>
<td>Aug 2009</td>
<td>36</td>
<td>8 (22.2) 18 (50) 10 (27.8)</td>
</tr>
<tr>
<td>Andrew Sentance§</td>
<td>Oct. 2006</td>
<td>May 2011</td>
<td>56</td>
<td>8 (14.3) 28 (50) 20 (35.7)</td>
</tr>
<tr>
<td>David Miles‡</td>
<td>June 2009</td>
<td>May 2015*</td>
<td>31</td>
<td>0 (0) 31 (100) 0 (0)</td>
</tr>
<tr>
<td>Adam Posen</td>
<td>Sep 2009</td>
<td>Aug 2012*</td>
<td>28</td>
<td>0 (0) 28 (100) 0 (0)</td>
</tr>
<tr>
<td>Martin Weale§</td>
<td>Aug 2010</td>
<td>July 2013*</td>
<td>17</td>
<td>0 (0) 10 (58.8) 7 (41.2)</td>
</tr>
<tr>
<td>Ben Broadbent§</td>
<td>June 2011</td>
<td>May 2014*</td>
<td>7</td>
<td>0 (0) 7 (100) 0 (0)</td>
</tr>
</tbody>
</table>

Table continues...
Figure 1: MPC members’ votes modelled as a Tempered Ordered Probit (TOP) model
Table 2: Estimation Results - All Models

<table>
<thead>
<tr>
<th>Equation</th>
<th>Variables</th>
<th>OP</th>
<th>IOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>OP Equation</td>
<td>$\pi_{Dev,t}$</td>
<td>0.195*** (0.025)</td>
<td>0.590*** (0.070)</td>
</tr>
<tr>
<td></td>
<td>$GAP_t$</td>
<td>0.055 (0.052)</td>
<td>0.150 (0.088)</td>
</tr>
<tr>
<td></td>
<td>$\mu_0$</td>
<td>-0.915*** (0.041)</td>
<td>-0.633*** (0.074)</td>
</tr>
<tr>
<td></td>
<td>$\mu_1$</td>
<td>1.103*** (0.046)</td>
<td>1.014*** (0.083)</td>
</tr>
</tbody>
</table>

Splitting Equation

| Constant        | -         | 1.300*** (0.464)  |
| TYPE            | -         | -0.546*** (0.129) |
| FTSE            | -         | 0.310*** (0.065)  |
| $\pi_\sigma$    | -         | 0.332 (0.067)     |
| GAP$\sigma$     | -         | -0.381*** (0.078) |
| IR              | -         | 0.904*** (0.139)  |
| NRI$^+$         | -         | -0.312*** (0.095) |

| AIC             | 2344.4447 | 1928.2760         |
| BIC             | 2365.7479 | 1986.8599         |
| CAIC            | 2369.7479 | 1997.8599         |
| LogL            | -1168.2224 | -953.13800         |

| OP Equation     | $\pi_{Dev,t}$ | 0.596*** (0.081)  | 0.905*** (0.085)  |
|                 | $GAP_t$       | 0.361*** (0.134)  | 0.157             |
|                 | $\mu_0$       | -0.613*** (0.107) | -0.597*** (0.153) |
|                 | $\mu_1$       | 0.606*** (0.194)  | 0.661*** (0.215)  |

| OP random parameters | $\sigma^2_\pi$ | - | 0.412*** (0.058) |
|                     | $\sigma^2_{\text{GAP}}$ | - | 0.288*** (0.145) |

<table>
<thead>
<tr>
<th>Splitting Equations</th>
<th>Down</th>
<th>Up</th>
<th>Down</th>
<th>Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.851 (1.435)</td>
<td>1.968 (0.604)</td>
<td>1.930 (1.198)</td>
<td>3.405 (0.910)</td>
</tr>
<tr>
<td>TYPE</td>
<td>-0.783*** (0.189)</td>
<td>-0.154 (0.131)</td>
<td>-1.053*** (0.318)</td>
<td>-0.586*** (0.323)</td>
</tr>
<tr>
<td>FTSE</td>
<td>1.245*** (0.231)</td>
<td>-0.192 (0.070)</td>
<td>1.450*** (0.263)</td>
<td>-0.317*** (0.116)</td>
</tr>
<tr>
<td>$\pi_\sigma$</td>
<td>-0.176 (0.144)</td>
<td>0.350*** (0.066)</td>
<td>-0.213*** (0.149)</td>
<td>0.225*** (0.094)</td>
</tr>
<tr>
<td>GAP$\sigma$</td>
<td>-0.125 (0.094)</td>
<td>-0.552*** (0.099)</td>
<td>-0.134*** (0.053)</td>
<td>-0.549*** (0.074)</td>
</tr>
<tr>
<td>IR</td>
<td>0.973*** (0.306)</td>
<td>0.723*** (0.188)</td>
<td>1.016*** (0.198)</td>
<td>0.989*** (0.218)</td>
</tr>
<tr>
<td>NRI$^+$</td>
<td>-0.6905*** (0.1206)</td>
<td>0.398* (0.213)</td>
<td>-0.653*** (0.125)</td>
<td>0.385*** (0.205)</td>
</tr>
</tbody>
</table>

Random effects

| $\sigma^2_{\text{down}}$ | - | 0.466*** (0.062) |
| $\sigma^2_{\text{up}}$   | - | 1.162*** (0.208) |

| AIC             | 1860.0843 | 2536315.1       |
| BIC             | 1953.9489 | 2536432.3       |
| CAIC            | 1973.9489 | 2536454.3       |
| LogL            | -912.04216 | -1268135.6     |

*Standard errors in parentheses.
***/*/** Denotes two-tailed significance at one / five / ten percent levels.
Table 3: TOP Estimates: Marginal Effects

<table>
<thead>
<tr>
<th>Ease</th>
<th>No Change</th>
<th>Tighten</th>
<th>No Change Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>OP equation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi_{Dev,t}$</td>
<td>$-0.179^{***}$</td>
<td>$0.1244^{***}$</td>
<td>$0.05551^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.01801)$</td>
<td>$(0.02264)$</td>
<td>$(0.01453)$</td>
</tr>
<tr>
<td>GAP$_t$</td>
<td>$-0.03115^{***}$</td>
<td>$0.02154^{***}$</td>
<td>$0.009614^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.02569)$</td>
<td>$(0.01729)$</td>
<td>$(0.00906)$</td>
</tr>
<tr>
<td>Splitting Equation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TYPE</td>
<td>$-0.2099^{***}$</td>
<td>$0.2323^{***}$</td>
<td>$-0.02239^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.07567)$</td>
<td>$(0.07764)$</td>
<td>$(0.01470)$</td>
</tr>
<tr>
<td></td>
<td>$0.2889^{***}$</td>
<td>$-0.2767^{***}$</td>
<td>$-0.01212^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.04587)$</td>
<td>$(0.04826)$</td>
<td>$(0.006495)$</td>
</tr>
<tr>
<td>GAP$_\sigma$</td>
<td>$-0.02680^{***}$</td>
<td>$0.04778^{***}$</td>
<td>$-0.02098^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.01099)$</td>
<td>$(0.01161)$</td>
<td>$(0.008145)$</td>
</tr>
<tr>
<td>IR</td>
<td>$0.2024^{***}$</td>
<td>$-0.2402^{***}$</td>
<td>$0.03779^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.04430)$</td>
<td>$(0.04184)$</td>
<td>$(0.01224)$</td>
</tr>
<tr>
<td>NRI$^+$</td>
<td>$-0.1301^{***}$</td>
<td>$0.1154^{***}$</td>
<td>$0.01471^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.02379)$</td>
<td>$(0.02267)$</td>
<td>$(0.007528)$</td>
</tr>
</tbody>
</table>

$^a$Standard errors in round (·) brackets; $^{\circ}$Denotes internal/external member.

$^{**/**/***}$Denotes two-tailed significance at one / five / ten percent levels.

Figure 2: MPC members’ votes modelled as a Tempered Ordered Probit (TOP) model
4 Conclusions

This paper empirically accounts for the stylized facts of voting on the policy rate associated with members of the Bank of England Monetary Policy Committee. We achieve this through combining a set of binary propensity to change equations or with an economic conditions equation. Estimation is performed using a discrete-choice framework, such that a new statistical model, the *Tempered Ordered Probit*, is proposed. It is likely that such a model may of use in numerous applied settings where the researcher is faced with modeling ordered discrete data with a disproportionately large occurrence of one of the middle outcomes. We have also demonstrated that the TOP model can be construed as a specification test of the MIOP model. Utilizing the panel nature of our data, unobserved effects were conditioned in each of the implicit underlying structural equations. We find substantial evidence that while external and internal members of the MPC react differently to the economic environment (via the economic conditions equation) the MPC is still characterized by individual voter heterogeneity and asymmetric attitudes towards for instance, economic uncertainty (via the set of inertia equation). Put another way, our findings suggest that members are characterized by quite different propensities to leave the interest rate unchanged, which in turn represents *asymmetric* attitudes towards, for instance, economic uncertainty and the changeability of interest rates. The inclusion of a set of inertia equations captures an important aspect of voting behavior which heretofore has not been modelled using discrete choice methods. Future work will to apply the TOP model to other committees for which voting data is available, such as Executive Board of the Swedish Riksbank and the United States FOMC.
References


